# DECO THEORY - BUBBLE MODELS

*This page describes principles and theories about bubble generation and bubble growth in the scuba divers body and about the effect of bubble formation on decompression and decompression sickness (DCS, bends) in scuba diving. Whereas classical (neo-)Haldane theories are mainly empirical and only take dissolved gas into account, bubble theories intend to give a physical explanation of the effects of bubbles on decompression. Bubble theories take dissolved and free gas into account. Especially the Varying Permeability Model (VPM) and Reduced Gradient Bubble Model (RGBM) give good explanation.*

## History

In classic decompression theory according to Haldane and successors a certain amount of supersaturation of the divers tissue with dissolved inert gas is allowed. The divers tissue is divided in a number of hypothetical tissue compartments. A certain limit (M-value) is associated with each compartment to supersaturation levels of dissolved inert gas in the compartment (tissue tension). This theory suggests efficient decompression by pulling the diver as close to the surface as possible with constraint that in all tissue compartments the supersaturated tissue tension remains within the limits. By pulling the diver as close to the surface the pressure gradient between the supersaturated tissue tension and the pulmonary (or arterial) gas is maximized. This enhances the elimination of the excess gas in the tissue. This theory is mainly empirical and based on experiment. At the moment most diving tables and computers are based on this theory.

Since the early days, diving has become more sophisticated by diving deeper and longer, the use of other breathing mixtures, etc. Some tech divers have made their own adaptations to the decompression schedules by inserting depression stops at greater depth ('deep stops', sometimes called 'Pyle stops' after Richard Pyle). These divers report feeling better when using these deep stops. This suggests that classic decompression theory fails in some situations and cannot be extrapolated to every diving situation. In order to gain insight in the principles of decompression, forming of bubbles during decompression has been studied for the last three decades. This has resulted in new theories like the Varying Permeability Model (VPM) by Yount et al. and the Reduced Gradient Bubble Model (RGBM). Bubble theories do not only take into account the dissolved gas (like the Haldane models), but also the free gas in the divers body. In this chapter we will have a look at some features of bubble theory. Lots of mathematics will be presented. The most important equations however, will be highlighted.

## Bubbles and surface tension

Consider a small air bubble in a glass of water. For the moment we neglect the solubility of the air in water. The small amount of air within the bubble is surrounded by a surface. The surface consists of water molecules which are unbound to one side. An unbound molecule represents more energy than a molecule which is completely surrounded by other water molecules. A *surface tension* $\gamma$ is associated with this surface between air and water. The surface tension is the amount of energy per unit of surface area and is expressed in $J/m^2$ or $N/m$.
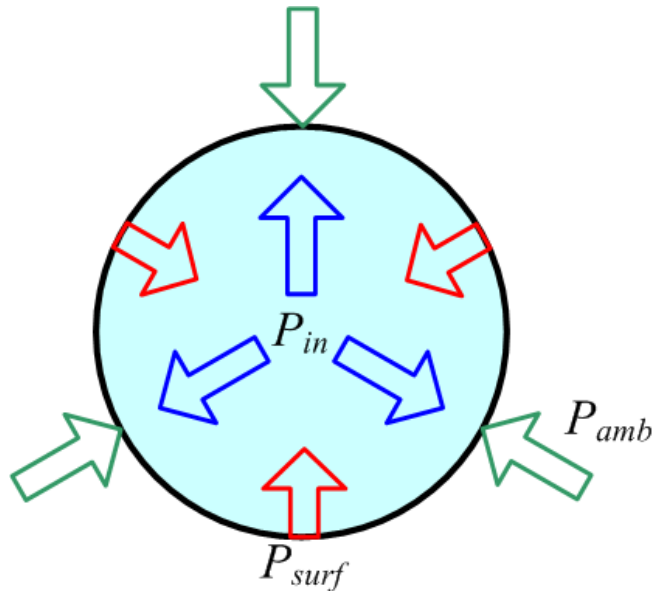
Figure 1: In equilibrium the internal pressure in the bubble is equal to the sum of the ambient pressure and the skin pressure due to the surface tension

A system will always try to minimize energy. Surface tension tends to minimise the bubble's surface. Hence, a bubble tends to collapse. However, collapsing a bubble decreases its volume. This will increase the gas pressure in the bubble (Boyle's law), until equilibrium is established: the internal pressure compensates the surface tension. The internal pressure due to the ambient pressure and surface tension is given by the Laplace equation:

$$P_{in} = P_{amb} + P_{surf} = P_{amb} + \frac{2\gamma}{r} \tag{1}$$

| | |
|---|---|
| $r$ | Radius of the bubble in $m$ |
| $\gamma$ | Surface tension in $joule/m^2$ of $N/m$. The surface tension of water at 273 K is 0.073 $N/m$. |
| $P_{in}$ | Pressure inside the bubble in $N/m^2 = 10^{-5} bar$ |
| $P_{amb}$ | Ambient pressure in $N/m^2 = 10^{-5} bar$ |
| $P_{surf}$ | Pressure due to the surface tension in $N/m^2 = 10^{-5} bar$ |

From this equation we learn that the smaller the bubble, the higher the pressure inside. You can experience the radius dependency of the pressure by trying to blow a balloon (bubble principles perfectly apply to a balloon up to the point where the balloon explodes). To get the first blow of air into the balloon (small radius) is a hell of a job, whereas it becomes easier if the balloon becomes larger.

## Bubbles and diffusion

When we have a bottle of beer things get a bit more complicated (usually the opposite holds, but when we look at the bubbles it might be). Bubbles in beer contain Carbon Dioxide. There is also Carbon Dioxide in solution in the beer. Carbon Dioxide can diffuse from the solution into the bubble or vice versa, depending on the partial pressure of the Carbon Dioxide in solution and in the bubble. If we assume that the bubble consist of only Carbon Dioxide, the Carbon Dioxide pressure in the bubble is given by equation 1 and depends on the radius of the bubble.

We define the partial pressure of the Carbon Dioxide in solution in the beer to be $P_t$ (if we regard the bottle of beer as a primitive model for a diver, we could call it 'tissue tension'). If the bottle is closed, the partial pressure of the Carbon Dioxide in solution $P_t$ is in equilibrium with the ambient pressure $P_{amb}$, which is the pressure in the bottle. If we assume there is only Carbon Dioxide gas in the (closed) beer bottle, the beer is saturated with Carbon Dioxide and $P_t$ will be equal to $P_{amb}$ (we can neglect hydrostatic pressure). The pressure in the bubble $P_{in}$ will be higher than $P_t$ due to the surface tension. Gas from within the bubble will diffuse into solution and the bubble will collapse. So every bubble will collapse eventually due to this gradient $P_{in} - P_t$. This is why in a closed bottle of beer there are no bubbles and there is no foam. However, if we open the bottle things will be different. The ambient pressure will drop, whereas the value of $P_t$ remains the same, at least for the moment. In this case $P_t$ is larger than $P_{amb}$: the beer is supersaturated with Carbon Dioxide.

Given an ambient pressure $P_{amb}$ and the partial pressure $P_t$ of the Carbon Dioxide in solution, there is a critical bubble radius $r^{min}$ at which the pressure inside the bubble $P_{in}$ equals $P_t$. The critical radius can be found by substituting $P_{in}$ by $P_t$ in equation 1:

$$r^{min} = \frac{2\gamma}{P_t - P_{amb}} \qquad (2)$$

For bubbles which size exceeds this critical size the pressure $P_{in}$ in the bubble is smaller than the partial pressure $P_t$ of the Carbon Dioxide in solution. Carbon Dioxide will diffuse from solution into the bubble. The bubble will grow. For bubbles smaller than the critical size, the opposite holds: gas from the bubble diffuses into solution and the bubble shrinks until it collapses completely. Bubbles at the critical size are in equilibrium, though it is an unstable equilibrium. This is depicted in Fig. 2.
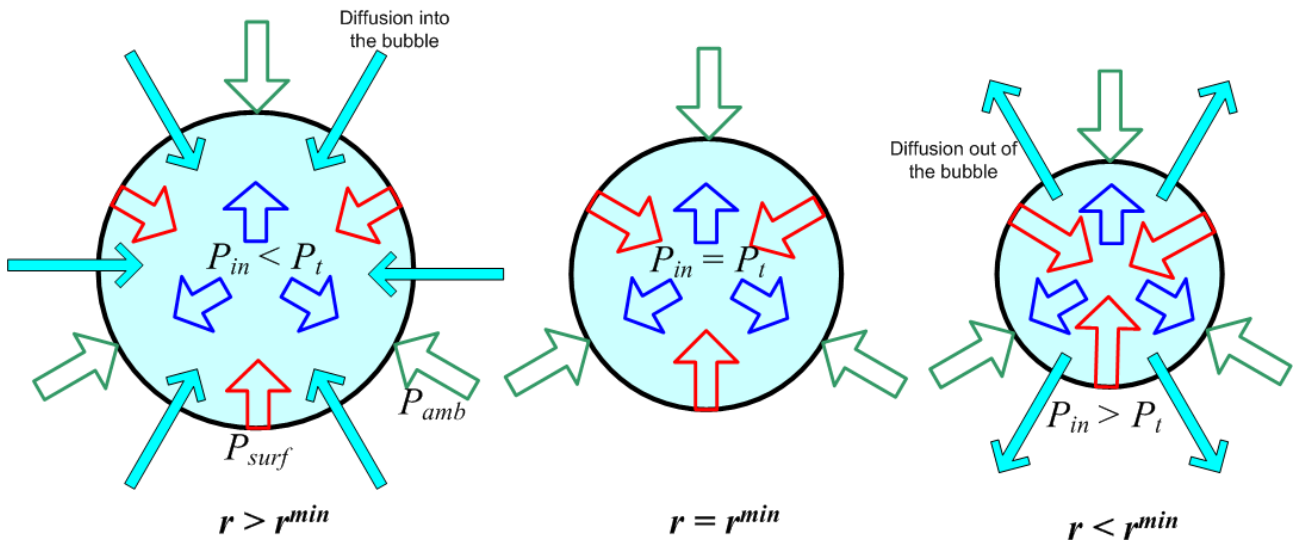


Figure 2: Bubble growth dependens on the radius

So every bubble with a radius larger than $r^{min}$ will start to grow. When we look at our opened bottle of beer we see bubbles becoming visible and heading for the surface, where they form foam. If you scrutinize a bubble you'll see that it grows during ascent. Its diameter might have doubled or tripled when it arrives at the surface. You might think this is due to Boyle's law. However it takes an ascent of several meters for a bubble to double its diameter. The growth of the bubble is due to the diffusion described above.

As an example, we can calculate critical radii for Spa Barisart Soda (6.4-8.0 g/l Carbon Dioxide). The pressure in the bottle specified by Spa is shown in next table (dependent on temperature). The partial pressure $P_t$ of the Carbon Dioxide in solution is roughly that value. If we open the bottle the ambient pressure $P_{amb}$ drops to 1 bar, whereas the partial pressure $P_t$ remains at the high value. Using equation 2 we can calculate the critical radius $r^{min}$.

| Temperature ($^{\circ}C$) | Pressure ($bar = 10^5 Pa$) | $r^{min}$ ($\mu m$) |
|:---:|:---:|:---:|
| 15 | 3 | 0.73 |
| 20 | 3.75 | 0.53 |
| 25 | 4.5 | 0.42 |
| 30 | 5.3 | 0.34 |
| 35 | 6 | 0.29 |
| 40 | 7 | 0.24 |

## The Varying Permeability Model

According to previous chapter, in a supersaturated situation any bubble exceeding a critical size $r^{min}$ will grow (and will disappear by floating to the surface) and any bubble smaller than this size will collapse. In a normal non-supersaturated situation, $r^{min}$ approaches infinity. Any bubble will collapse. So we do not expect any bubbles around after a while. You might expect that if no initial bubbles are around, there is no bubble to grow on supersaturating the liquid. The tensile strength of water is estimated on 1000 atm, making immense supersaturations possible, before bubbles (voids) are created. If no initial bubbles would be present in the water making up the diver, a diver could easily dive to a kilometer depth and pop up to the surface without any problems. In practice, this is not the case. Bubbles form on modest decompression as low as 1 atm. Here comes in the Varying Permeability Model (VPM). The VPM was initially defined by Yount et al. [2] in order to give a quantitative explanation on the formation of bubbles in decompressed gelatin [1] (as model for divers tissue). Later on, they showed this model can be used to calculate dive tables as well [3], [4]. In next paragraphs we will have a look at the gelatin theory. Later on we will apply the theory to diving.
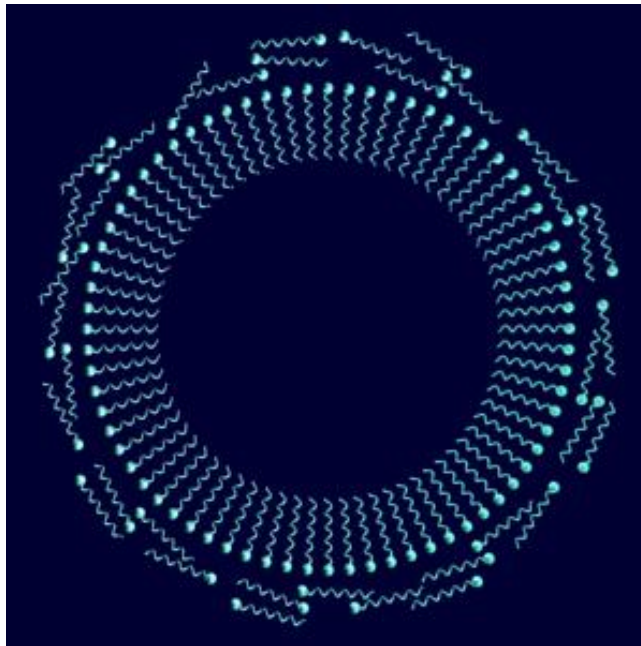


Figure 3: Skins of varying permeability are the base of the VPM

### The gelatin experiments

Experiments on gelatin have been performed, by David Yount and other researchers [1]. The advantage of gelatin over water is that any bubble appearing during decompression gets trapped and won't flow to the surface. In this way they can be observed and counted. Yount applied the rudimentary pressure of Figure 4 to gelatin samples: Gelatin samples were made at ambient pressure Pamb=P0 of 1 atm. The samples were rapidly compressed in a 100
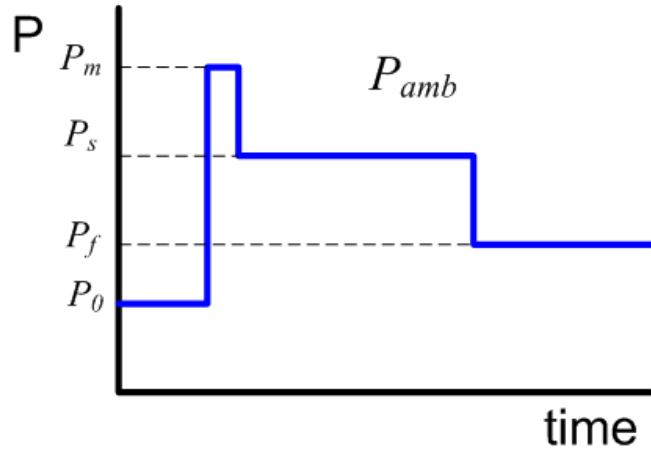
Figure 4: Pressure schedule applied to the gelatin samples by Yount

According to the VPM, in aqeous media like water and gelatin stable gaseous cavities are present. They are called nuclei. Radii range from a few 1/100 m up to around 1 m. Any nucleus in water larger will flow to the surface and disappear. Whereas an ordinary bubble with these radii would collapse under normal conditions (no supersaturation), these nuclei appear to be exceptionally stable and have a long life. Yount proposed this stability is due to an elastic skin made up of surfactant, as shown schematically in Figure 3. Surfactant consists of (hydrophobic) surface active molecules, which are aligned. During the compression stage, these skins are permeable for gas up to a pressure of around 8 atm. Diffusion through the skin takes place. The pressure $P_{in}$ of the gas in the nucleus is equal to the dissolved gas tension $P_t$ in the surrounding liquid. Above this pressure, the skin becomes impermeable. Upon decompressing (reducing the ambient pressure) the skins are regarded permeable. The skin gives rise to a 'surface compression' $\Gamma$ which opposes the regular surface tension $\gamma$ of the water/air surface, as shown in Figure 5:

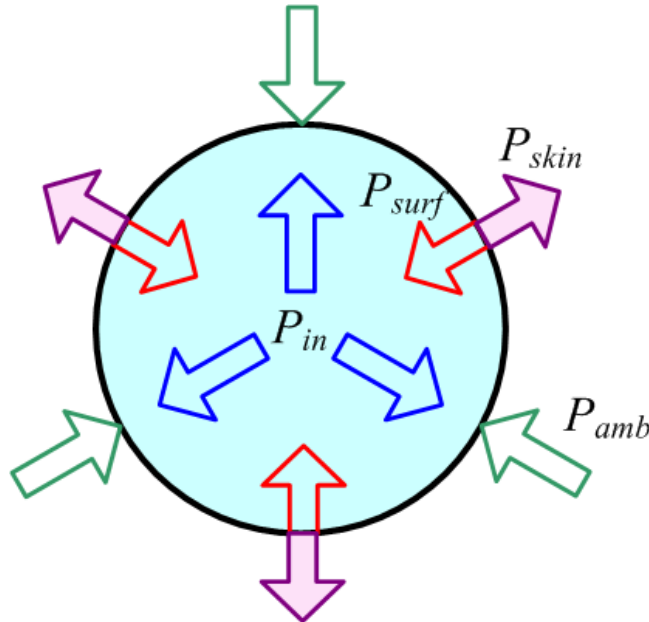$$P_{in} + \frac{2\Gamma}{r} = P_{amb} + \frac{2\gamma}{r} \tag{3}$$



Figure 5: Pressures acting on the surface of the bubble

The skin tension $\Gamma$ is not constant but ranges from 0 to a maximum $\gamma_c$, which is called the 'crumbling compression'. The idea is that small variations of the size of the nucleus can be supported by varying the distance between the molecules in the skin. This gives rise to varying $\Gamma$. This situation is described by equation 3 and is referred to as the small-scale situation. In this equilibrium situation and in the permeable region, due to

diffusion the internal pressure $P_in$ is equal to the tension $P_t$. In the samples (no hydrostatic pressure, 100% Nitrogen) $P_t$ equals Pamb. So $P_{in} = P_t = P_{amb}$. In this situation $\Gamma$ equals $\gamma$, according to equation 3.

Upon compressing and decompressing, variation of the size of the nucleus becomes to large to be supported by varying distances between molecules. Surfactant molecules have to be expelled from or taken up into the skin in order to compensate for the area decrease resp. increase of the nucleus. This is schematically shown in Figure 6. The skin is surrounded by an amount of surfactant, which is not part of the skin. This amount acts as a reservoir, taking up or supplying surfactant molecules from or to the skin. The reservoir molecules are not aligned and cannot support a pressure gradient. $\Gamma$ takes its crumbling value $\gamma_c$ in this large-scale situation. Yount proposes two derivations of the VPM [2]: one from a thermodynamic point of view and one from a mechanical point of view.
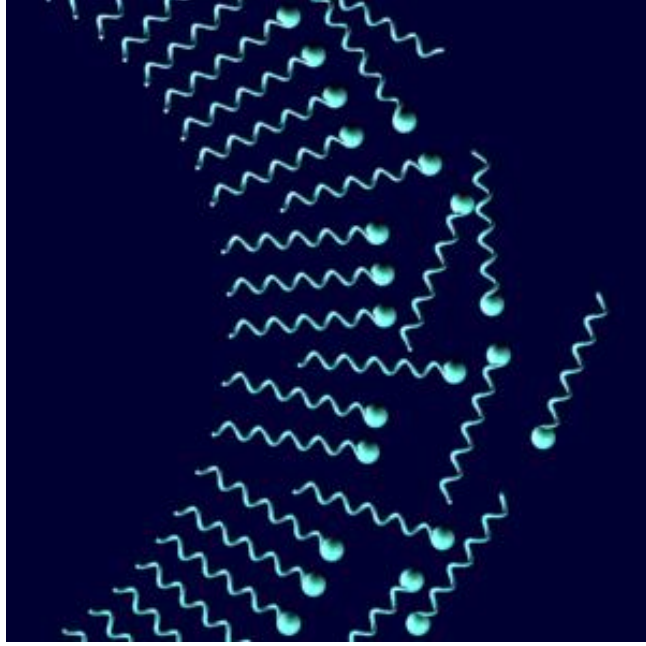


Figure 6: The large-scale situation: variation in the size of the nucleus result in expelling molecules from the skin

In the original sample there is a initial distribution of nuclei with radii distributed according to some function $f(r_0)$. (The '0' in $r_0$ refers to the initial situation). On applying the pressure schedule, it is assumed that all nuclei with a radius larger than some minimal initial radius $r_0^{min}$ will grow into bubbles. The number of bubbles $N$ that occur is given by the integration of $f(r_0)$ from $r_0^{min}$ to infinity.

$$N = \int_{r_0^{min}}^{\infty} f(r_0)\mathrm{d}r_0 \tag{4}$$

Applying this theory to a diver, it might be assumed that the severity of Decompression Sickness (DCS) might be related to this number of bubbles, which occur after decompression. Hence, $r0min$ becomes an indication for the severity of DCS.

It is assumed that no nuclei are extinguished or created during application of the pressure schedule. Furthermore it is assumed that the ordering of nuclei is preserved: if one nucleus is larger than an other one, this is still true after a pressure change (ordering hypothesis). At the end of the pressure schedule there is a new distribution of radii $g(r_f)$ and a new radius $r_f^{min}$ above which all nuclei will grow into bubbles. Note: a nucleus with radius $r_0^{min}$ ends up as a nucleus with radius $r_f^{min}$ after application of the presure schedule. The aim of next VPM calculations is

1. To define the allowed number of bubbles by defining $r_0^{min}$

2. To find a relation between $r_f$ and $r_0$ (and hence between $r_f^{min}$ and $r_0^{min}$)

3. To find the relation based on $r_f^{min}$ which governs the bubble formation on decompression.

4. To calculate the resulting restricting relations for the pressure schedule, given the value of $r_0^{min}$ and hence, the number of resulting bubbles after application of the pressure schedule.

## Thermodynamic equilibrium

From a thermodynamic point of view the left-hand side of equation 3 represents the skin pressure $P_S$:

$$P_S = P_{in} + \frac{2\gamma_c}{r} \tag{5}$$

$\Gamma$ has been replaced by the large-scale value $\gamma_c$. Similarly the right hand term of equation 3 represents the reservoir pressure $P_R$:

$$P_R = P_{amb} + \frac{2\gamma}{r} \tag{6}$$

In the large-scale situation transport of surfactant is not described by setting $P_R$ equal to $P_S$ but by the requirement that the electrochemical potential in the skin and reservoir are equal. The electrochemical potential $\xi$ is given by

$$\xi = \mu + kT \ln(\rho) + pv + Ze\ \psi \tag{7}$$

$\xi$     Electrochemical potential
$\mu$     Pure chemical potential
$k$     Bolzmann contant
$T$     Absolute temperature in K
$\rho$     Molecular concentration or number density
$p$     Static pressure
$v$     Active volume occupied by one surfactant molecule
$Ze$     Effective charge of one surfactant molecule
$\psi$     Electrostatic potential

In the reservoir we have

$$\xi_R = \mu_R + kT \ln(\rho_R) + P_R v + (Ze\ \psi)_R \tag{8}$$

and in the skin we have

$$\xi_S = \mu_S + kT \ln(\rho_S) + P_S v + (Ze\ \psi)_S \tag{9}$$

Requiring $\xi_R$ is equal to $\xi_S$ and substituting $P_S$ and $P_R$ by the values in equation 5 resp 6 results in:

$$P_{in} + \frac{2\gamma_c}{r} - \beta = P_{amb} + \frac{2\gamma}{r} \tag{10}$$

in which

$$\beta = \frac{1}{v} \left[ kT \ln \frac{\rho_R}{\rho_S} + (\mu_R - \mu_S) + (Ze\ \psi)_R - (Ze\ \psi)_S \right] \tag{11}$$

Equations 10 and 11 can be used to calculate the changes in radii after applying a pressure step. We have a look what happens when applying the pressure schedule of Figure 4 to the sample. At the beginning of the pressure schedule $P_{amb} = P_0$. The pressure of the gas in the nucleus is

$$P_{in} = P_{t0} = P_{amb} = P_0 \tag{12}$$

Just before compression equation 10 is:

$$P_0 + \frac{2\gamma_c}{r_0} - \beta_0 = P_0 + \frac{2\gamma}{r_0} \tag{13}$$

After the pressure rise to the pressure $P^*$ where the skins becomes impermeable equation 10 reads:

$$P_0 + \frac{2\gamma_c}{r^*} - \beta^* = P^* + \frac{2\gamma}{r^*} \tag{14}$$

Substracting equation 14 from 13, assuming $\beta_0 = \beta^*$ and rewriting it a bit result in:

$$2(\gamma_c - \gamma)\left[\frac{1}{r^*} - \frac{1}{r_0}\right] = P^* - P_0 \tag{15}$$

We continue to compress rapidly from $P^*$ to $P_m$. Since the skin is not permeable now, the pressure in the nucleus varies with its radius according to Boyle's law ($PV = P4/3\pi r^3 = constant$):

$$P_{in} = P_t^* \frac{r^{*3}}{r^3} \tag{16}$$

In this equation $r^*$ is the radius of the nucleus at the beginning of the impermeable process. $P_t^*$ is the corresponding dissolved gas tension, which is equal to $P_0$. So after the compression to $P_m$ we have:

$$P_0\left(\frac{r^*}{r_m}\right)^3 + \frac{2\gamma_c}{r_m} - \beta_m = P_m + \frac{2\gamma}{r_m} \tag{17}$$

Assuming $\beta_m = \beta^* = \beta_0$, subtracting equation 17 from 14 and rewriting a bit we end up with:

$$2\left(\gamma_c - \gamma\right)\left(\frac{1}{r_m} - \frac{1}{r^*}\right) = P_m - P^* + P_0\left[1 - \left(\frac{r^*}{r_m}\right)^3\right] \tag{18}$$

We now have relations (equation 15 and 18) between the radius $r_m$ of the nucleus after compressing and the radius $r_0$ prior to compression. The saturation phase ($P_{amb} = P_s$) that follows saturates the liquid so that finally the dissolved gas tension $P_{ts} = P_s$. We might expect that the radius of the nucleus increases to its original value (see the the note in the 'Mechanical Equilibrium' section). However, this has not been observed. So we assume

$$r_s = r_m \tag{19}$$

In fact the radius restores quite slowly, but for the moment equation 19 holds. Prior and after decompressing (which is fully permeable) equation 10 reads:

$$P_s + \frac{2\gamma_c}{r_s} - \beta_s = P_s + \frac{2\gamma}{r_s} \tag{20}$$

resp.

$$P_s + \frac{2\gamma_c}{r_f} - \beta_f = P_f + \frac{2\gamma}{r_f} \tag{21}$$

Assuming $\beta_f = \beta_s$ (not equal to $\beta_m$, subtracting equation 21 from 20 and rewriting a bit result in:

$$2\left(\gamma_c - \gamma\right)\left(\frac{1}{r_f} - \frac{1}{r_s}\right) = P_f - P_s \tag{22}$$

So we now have a relation between all radii of the nucleus during the entire profile. A nucleus with radius $r_0$ ends up as a nucleus with radius $r_f$ through a number of stages ($r^*$, $r_m$, $r_s$) defined by the relations 15, 18, 19 and 22.

We now define the criterion for bubble formation, which is given by the Laplace equation 1:

$$P_{in} - P_{amb} = P_s - P_f \geq \frac{2\gamma}{r_f^{min}} \tag{23}$$

There is no reference to $\Gamma$ or $\gamma_c$ in this equation. We assume the skin of the nucleus to be permeable. So the skin does not restrict bubble formation: gas simply flows through the skin and forms a gas shell outside the skin. If the skin should not be permeable as has been proposed by others some tearing strenght or tearing tension $\Gamma = -\gamma_T$ is introduced. The bubble forming equation becomes:

$$P_{in} - P_{amb} = P_s - P_f \geq \frac{2\,(\gamma + \gamma_T)}{r_f{}^{min}} \tag{24}$$

By combining equation 15, 18, 19, 22 and 23 we find the VPM equations.

For the ever-permeable region $P_m \leq P^*$:

$$P_{ss}{}^{min} = 2\gamma \frac{\gamma_c - \gamma}{r_0{}^{min}\gamma_c} + P_{crush}\frac{\gamma}{\gamma_c} \tag{25}$$

For the permeable-impermeable-permeable situation $P_m > P^*$ we find:

$$P_{ss}{}^{min} = 2\gamma \frac{\gamma_c - \gamma}{r_0{}^{min}\gamma_c} + \left[ P_m - P_0 \left( \frac{r^*}{r_m} \right)^3 \right] \frac{\gamma}{\gamma_c} \tag{26}$$

In these equations we have the crushing pressure

$$P_{crush} = (P_{amb} - P_t)_{max} = P_m - P_0 \tag{27}$$

and the supersaturation pressure

$$P_{ss}{}^{min} = (P_t - P_{amb})_{max} = P_s - P_f \tag{28}$$

Equation 26 can be written as:

$$P_{ss}{}^{min} = 2\gamma \frac{\gamma_c - \gamma}{r_0{}^{min}\gamma_c} + \frac{\gamma}{\gamma_c}\,(P^* - P_0) + \frac{\gamma}{\gamma_c}\frac{1}{\left(1 + \frac{r^\sim}{B}\right)}\,(P_m - P^*) \tag{29}$$

The parameters $r^\sim$ and $B$ are defined as:

$$r^\sim = r^* \frac{r^*}{r_m} \tag{30}$$

$$B = \frac{2\,(\gamma_c - \gamma)}{P_0 \left( \frac{r^*}{r_m} + 1 + \frac{r_m}{r^*} \right)} \tag{31}$$

**Mechanical equilibrium**

The other way the VPM is derived is by looking from a mechanical point of view. Changes in nuclear radius can be calculated by the equation proposed by Love, which reads (in the VPM form, [2]):

$$2(\Gamma - \gamma)\frac{\partial r}{r^2} = \partial P_{in} - \partial P_{amb} \tag{32}$$

In the 'permeable' region of the VPM, $P_{in}$ remains constant and equal to $P_t$. Here $\partial P_{in}$ is 0. For large-scale variations in the 'permeable' region of VPM equation 32 reads

$$2(\gamma_c - \gamma)\frac{\partial r}{r^2} = \partial P_{amb} \tag{33}$$

In the 'impermeable' region, $P_{in}$ is given by equation 16. Differentiated it reads:

$$\partial P_{amb} = -\left(3P_t{}^*\frac{r^{*3}}{r^4}\right)\partial r \tag{34}$$

For the large-scale variations in the impermeable region, equation 32 reads

$$\left(2\left(\gamma_c - \gamma\right) + 3P_t{}^*\frac{r^{*3}}{r^2}\right)\frac{\partial r}{r^2} = \partial P_{amb} \tag{35}$$

Together with the Laplace equation 23 and the assumption of equation 19, equation 33 and 32 can be used (by integrating) to derive the VPM equations 25 and 26: equation 15 and 22 can be obtained by integrating equation 33, equation 18 can be obtained by integrating equation 35. The derivation is given in [2].

Note: During the compression phase $\partial P_{in}$ is zero in equation 32(24). Assuming the pressure schedule takes place in the permeable region, integration of 32 from $P_{amb} = P_0$ to $P_m$ results in:

$$2\left(\gamma_c - \gamma\right)\left(\frac{1}{r_m} - \frac{1}{r_0}\right) = P_m - P_0 \tag{36}$$

During the saturation phase that follows $\partial P_{amb}$ is zero in equation 32. Integration of 32 from $P_in = P_0$ to $P_s$ results in:

$$2\left(\gamma_c - \gamma\right)\left(\frac{1}{r_s} - \frac{1}{r_m}\right) = P_0 - P_s \tag{37}$$

Adding equation 37 to 36 results in $r_s = r_0$, assuming $P_m = P_s$ (in fact in the non-permeable situation or a situation in which $P_m \neq P_s$ we could derive the same result, though it takes some more derivation). This suggests that the nucleus is fully restored to the original size during saturation. The effect of the crushing is lost in this situation. However, this is in sharp disagreement with experiment. Further indication is given by special cases in which a pressure spike is present at the start of the schedule (Figure 4), so that $P_m < P_s$. In this cases the bubble count only depend on $P_{ss}$ and $P_{crush}$ and not on $P_s$! Hence, the assumption defined by equation 19 is made: $r_s = r_m$.

**Equilibrium considerations**

In this section we will consider some implications from the equilibria discussed above.

Rewriting equations 13, 14, 17, 20 and 21 gives us:

$$\beta_0 = \frac{2\left(\gamma_c - \gamma\right)}{r_0} \tag{38}$$

$$\beta^* = \frac{2\left(\gamma_c - \gamma\right)}{r^*} - \left(P^* - P_0\right) \tag{39}$$

$$\beta_m = \frac{2\left(\gamma_c - \gamma\right)}{r_m} - \left(P_m - P_0\left(\frac{r^*}{r_m}\right)^3\right) \tag{40}$$

$$\beta_s = \frac{2\left(\gamma_c - \gamma\right)}{r_s} \tag{41}$$

$$\beta_f = \frac{2\left(\gamma_c - \gamma\right)}{r_f} - \left(P_f - P_s\right) \tag{42}$$

(We recall our assumptions that $\beta_0 = \beta_m = \beta^*$ and $\beta_s = \beta_f$.) According to equation 11 $\beta$ is independent of radius. However, according to equation 38 $\beta_0$ appears to be a function of $r_0$. Assuming that $\beta_0$ is constant for all nuclei at $P_{amb} = P_0$ we obtain an remarkable prediction that $\gamma_c$ increases with increasing $r_0$:

$$\gamma_c = \gamma + \frac{r_0 \beta_0}{2} \tag{43}$$

Another consideration stems from mechanical equilibrium: Small scale equilibrium is given by equation 3:

$$P_{in} + \frac{2\Gamma}{r} = P_{amb} + \frac{2\gamma}{r} \tag{44}$$

All properties of the skin and the reservoir are incorporated in the small scale skin compression $\Gamma$. The equation can be obtained from 10 by setting:

$$\frac{2\Gamma}{r} = \frac{2\gamma_c}{r} - \beta \tag{45}$$

Substituting the $\beta$ values of equation 38 to 42 in 45 results at the respective ambient pressure values $P_{amb} = P_0$, $P^*$, $P_m$, $P_s$ and $P_f$ in:

$$\Gamma_0 = \gamma \tag{46}$$

$$\Gamma^* = \gamma_c - \frac{\gamma_c - \gamma}{r_0} r^* \tag{47}$$

$$\Gamma_m = \gamma_c - \frac{\gamma_c - \gamma}{r_0} r_m \tag{48}$$

$$\Gamma_s = \gamma \tag{49}$$

$$\Gamma_f = ? \tag{50}$$

A plausible small scale/mechanical equilibrium criterion for bubble formation is that $\Gamma_f$ is less than or equal to zero. This results in:

$$\frac{2 (\gamma_c - \gamma)}{r_s} \geq \frac{2\gamma_c}{r_f} \tag{51}$$

Substituting this in equation 22 results in the Laplace equation 23 as used for the thermodynamic derivation.

Equation 46 to 50 shows that during the compression $\Gamma$ increases. During saturation $\Gamma$ 'relaxes' to its value prior to compression $\gamma$, keeping $r_m$ constant. During decompression $\Gamma$ drops to 0, the point at which bubble formation just starts.

## Consequences of the VPM relations

### Plotting $P_{ss}$ vs. $P_{crush}$

Most conveniently, equation 25 is plotted as $P_{ss}^{min}$ vs. $P_{crush}$. In these plots, $P_{ss}^{min} - P_{crush}$ pairs resulting in the same number of bubbles (and hence, the same DCS morbidity) form straight lines.

### Diver vs. gelatin

The VPM originally was developed to quantitatively explain bubble formation in gelatin during decompression [2]. The ultimate goal was to gain understanding of decompression sickness. To apply VPM to a diving situation it first was suggested that decompression sickness (DCS) symptoms were related to the number of bubbles. Say, severe symptoms occur at a number NDCS of bubbles in some tissue. Given the radial distribution $f(r_0)$, equation 4 defines a $r_0^{min}$. If all nuclei with a radius equal or larger than this radius grow into bubbles, we end up with NDCS bubbles (and some bad DCS). Given a dive to some depth resulting in an ambient gradient $P_{crush}$, equation 25 gives the maximum allowed gradient $P_{ss}^{min}$ resulting in the NDCS bubbles

**The VPM relations**

The VPM relations 25 and 26-28 define the maximum allowed gradient between the ambient pressure and the tissue tension. In other words: it defines the minimum allowed ambient pressure $P_{amb}$, given the tissue tension $P_t$. In a diving situation it defines the depth the diver is allowed to ascend given the tissue tension. The relation should be applied to each tissue compartment of the diver.

The initial compression (defining $P_{crush}$) is important for Pssmin. During this stage nuclei are crushed to a smaller size, making them less active in bubble formation. The secret lies in equation/assumption 19, which states that no regeneration of the bubble size takes place during saturation. It implies that a descent during a dive should be as quick as possible, the deepest part of the dive should be at the start of the dive and deeper dives should precede shallower dives in a repetitive dive situation. These facts have been empirically found during a century of decompression research.

**Not a 100% Nitrogen saturation dive**

The derivation of the VPM assumed 100% Nitrogen and fully saturated gelatin. If we apply the equations to a non-saturating diving situation in which the Nitrogen fraction is less than 100% (for example air, containing 79% Nitrogen), the VPM equations 25 and 26-28 can be regarded as a conservative restriction to the dive profile.

# Applying VPM to diving

In this section we will apply the VPM to a diving situation and describe a method to generate diving tables. Whereas the VPM theory of previous sections applies to a special situation of fully saturated gelatin in a 100% Nitrogen atmosphere, situations during diving are different. The assumption that the severity of DCS is proportional to the absolute number of bubbles leads to very safe diving tables, not covering all of the conditions of modern diving tables and often leading to unacceptable long decompression periods. The VPM was reformulated, as described in this section, to fit it with conventional diving tables. Conventional diving tables were regarded as valid measurements. We will follow the derivation of Yount [3]. During this derivation we assume only one inert gas. Later on we will place remarks on using more inert gasses (Trimix, etc). Another assumption is the dive takes place in the 'permeable' region of the VPM.

**The reformulated VPM**

The derivation of the theory below is based on a number of more or less ad hoc assumptions. The most important assumptions concern the relationship between decompression symptoms and the amount of free gas (bubbles) in the divers tissue:

1. There is an amount of bubbles $N_{safe}$ which can be tolerated by the divers body, independent of all circumstances (like tissue tension, degree of supersaturation, etc). The initial critical radius corresponding to this number is $r_0^{min}$ (equation 4).

2. The actual number of bubbles $N_{actual}$ may be higher than $N_{safe}$ as long as the total volume $V$ of all free gas always remains below a critical value $V_{crit}$. This is called the 'critical-volume hypothesis'. A initial radius $r_0^{new}$ smaller than $r_0^{min}$ is associated with this number.

3. The volume of free gas $V$ inflates at a rate proportional to $P_{ss}\left(N_{actual} - N_{safe}\right)$, where $P_{ss}$ is the saturation $P_t - P_{amb}$.

The first of these assumption agrees with physiological studies, which state that the lungs are able to continue functioning as a trap for venous bubbles to a certain degree. From this assumption can be deduced that the rate at which the body can dissipate free gas by exchange in the lungs is proportional to both the supersaturation pressure $P_{ss}$ and $N_{safe}$.

The assumption defined by equation 19 is fine tuned according to observations: the radius $r_m$ slowly regenerates during saturation instead of remaining unchanged, as stated by equation 19. The regeneration is exponential, governed by a regeneration time constant $\tau_R$:

$$r_s\left(t_R\right) = r_m + \left(r_0 - r_m\right)\left(1 - e^{\frac{t_R}{\tau_R}}\right) \tag{52}$$

| | |
|---|---|
| $r_s$ | Nuclear radius just prior to ascent and decompression (m) |
| $r_m$ | Nuclear radius after compression by $P_{crush}$ (m) |
| $r_0$ | Nuclear radius before descent (m) |
| $t_R$ | Regeneration period: time from start of dive up to start of ascent and decompression (min) |
| $R$ | Regeneration time constant (min) |

If we wait long enough the crushed nucleus will end up with its initial radius prior to compression $r_0$.

In contrast with other decompression models, VPM takes the effect of other gasses (water vapor, Oxygen, Carbon Dioxide) into account in calculating the tissue tension:

$$P_{t\_total} = P_{inert\_gasses} + P_{other\_gasses} \tag{53}$$

| | |
|---|---|
| $P_{t\_total}$ | Tissue tension |
| $P_{inert\_gasses}$ | Sum of the partial pressures of the dissolved inert gasses |
| $P_{other\_gasses}$ | Pressure due to water vapor, Oxygen and Carbon Dioxide. Yount specifies a nearly constant value of 102 mm Hg (corresponding to 0.136 bar) for inspired partial Oxygen pressures up to 2 atm [5] |

The supersaturation is now defined as:

$$P_{ss} = P_t + P_{amb} \tag{54}$$

The reformulated VPM now consist of the following steps:

1. Specify the parameters defining the VPM: surface tension $\gamma$, the crumbling compression $\gamma_c$, the minimum initial radius $r_0^{min}$, the regeneration time constant $\tau_R$ and a composite parameter $\lambda$. The latter is related to the critical volume $V_{crit}$. The parameters are the same for each compartment.

2. Calculate the initial allowed supersaturation that is just sufficient to probe $r_0^{min}$ and that results in $N_{safe}$ bubbles. The equation for this is:

$$P_{ss}^{min} = 2 \frac{\gamma}{\gamma_c} \frac{\gamma_c - \gamma}{r_s} t_R \tag{55}$$

In fact, this is an enhanced equation 25, taking nuclear regeneration into account. In this equation the regenerated radius $r_s(t_R)$ is given by equation 52. Since the VPM parameters are the same for each tissue compartments, this initial allowed supersaturation gradient will be the same for each compartment.

3. Calculate a decompression profile, using this $P_{ss}^{min}$. The total decompression time defined by the profile is $t_D$.

4. Calculate a new allowed supersaturation gradient $P_{ss}^{new}$ using:

$$P_{ss}^{new} = \frac{1}{2} \left[ b + \left( b^2 - 4c \right)^{1/2} \right] \tag{56}$$

where

$$b = P_{ss}^{min} + \frac{\lambda \gamma}{\gamma_c \left( t_D + \frac{1}{k} \right)} \tag{57}$$

$$c = \left( \frac{\gamma}{\gamma_c} \right)^2 \frac{\lambda P_{crush}}{t_D + \frac{1}{k}} \tag{58}$$

In these equation is $k = ln(2)/\tau$, where $\tau$ is the half-time of the tissue compartment. This result in a larger allowed supersaturation gradient $P_{ss}^{new}$. Of course, this step is repeated for each tissue compartment.

5. Perform a number of iteration of step 3-4, until $t_D$ and $P_{ss}^{new}$ converge. Of course, occurrences of $P_{ss}^{min}$ are now substituted by $P_{ss}^{new}$.

After the iterations we end up with a more severe decompression profile and a $P_{ss}^{new}$ corresponding to a new initial critical radius $r_0^{new}$, which is smaller than $r_0^{min}$. This new radius results in a larger number of bubbles $N_{actual}$ and a maximum volume of free gas approaching $V_{crit}$.

**More inert gasses**

In some (tech) diving situations, other gas mixtures are used consisting of more than one inert gas (for example Trimix, containing Oxygen, Nitrogen and Helium). In next remarks we assume Helium and Nitrogen to be the inert gases.

1. For each gas, the VPM parameters should be specified. For each tissue compartment a half-time for each gas should be specified.

2. For each gas, the allowed supersaturation gradient should be calculated using the method in previous section. In this case the supersaturation gradient for Helium is $P_{ss\_He}$ and for Nitrogen is $P_{ss\_N}$

3. If $P_{t\_He}$ and $P_{t\_N}$ are the Helium and Nitrogen tissue tensions, the total tissue tension is given by:

$$P_{t\_total} = P_{t\_He} + P_{t\_N} + P_{other\_gasses} \tag{59}$$

4. The allowed supersaturation gradient is given by the weighted average:

$$P_{ss\_total} = P_{t\_total} - P_{amb} = \frac{P_{t\_He}P_{ss\_He} + P_{t\_N}P_{ss\_N}}{P_{t\_He} + P_{t\_N}} \tag{60}$$

**Derivation**

In this section we will derive the new VPM equations 56-58. The allowed supersaturation gradient $P_{ss}^{min}$ as given by equation 25, 26 and 55 can be applied to diving as a safe-ascent criterion. Whereas they can be derived directly from VPM, the derivation of $P_{ss}^{new}$ in equation 56-58 involves a number of ad hoc assumptions.

**Assumption 1**: The total volume of free gas in the divers body should never exceed a critical volume value $V_{crit}$ at any time $t$ (not during the dive, nor thereafter).

**Assumption 2**: The rate at which the free gas inflates is proportional to $P_{ss}(t)(N_{actual} - N_{safe})$. In this equation $Pss(t) = P_t(t) - P_{amb}(t)$.

Assumption 1 and 2 result in the decompression criterion:

$$\int_0^t P_{ss}(t)\,(N_{actual} - N_{safe})\,dt \leq \alpha V_{crit} \tag{61}$$

In this equation is $\alpha$ a proportionality constant. This criterion should hold for any $t$. To minimise the decompression time $t_D$, the $\leq$ sign is replaced by the $=$ sign.

**Assumption 3**: The actual number of bubbles $N_{actual}$ and the number of bubbles always allowed $N_{safe}$ are determined by the initial decompression stop and remain constant thereafter. The decompression criterion now reads:

$$\alpha V_{crit} = (N_{actual} - N_{safe}) \int_0^{t_{max}} P_{ss}(t)dt \tag{62}$$

In this equation $t_{max}$ is the value of $t$ at which the integral reaches the maximum value.

**Assumption 4**: The decompression profile is chosen so that $P_{ss}(t)$ remains at constant value $P_{ss}^{new}$ during the ascent period $t_D$ and decays exponentially to zero thereafter (at the surface). This is in agreement with Assumption 3: $P_{ss}(t)$ is always positive and never exceeds its initial value $P_{ss}^{new}$. This initial value is the maximum value defining $N_{actual}$. The latter remains constant thereafter. The exponential decay to zero is a conservative approximation: according to Yount&Lally [5] humans are 'inherently unsaturated' when equilibrated at atmospheric pressure by about 54 mm Hg (0.072 bar). Eventually, $P_{ss}(t)$ will become negative by this amount.

Due to Assumption 4 and the exponential decay, the integral of equation 62 reaches it maximum value in the limit as $t_{max}$ approaches $\infty$. The criterion for decompression now becomes:

$$\alpha V_{crit} = (N_{actual} - N_{safe}) \int_0^{t_D} P_{ss}^{new}(t)dt + \int_{t_D}^{\infty} P_{ss}^{new} e^{-k(t-t_D)} \tag{63}$$

In this equation is $k = ln(2)/\tau$ where $\tau$ is the tissue compartment halftime.

**Assumption 5**: The distribution of nuclei in humans is not known. An decaying exponential relation is assumed, observed in vitro:

$$N_{actual} = N_0 e^{-\frac{\beta_0 S r_0^{new}}{2kT}} \tag{64}$$

$$N_{safe} = N_0 e^{-\frac{\beta_0 S r_0^{min}}{2kT}} \tag{65}$$

$\beta_0$    VPN Constant
$N_0$    Normalisation constant
$S$    Constant area, occupied by one surfactant molecule in situ
$k$    Bolzmann contant
$T$    Absolute temperature in K

The decompression criterion can be rewritten:

$$P_{ss}^{new} = \frac{\alpha V_{crit}}{(N_{actual} - N_{safe})\left(t_D + \frac{1}{k}\right)} \tag{66}$$

where

$$(N_{actual} - N_{safe}) = N_0 \left[ e^{-\frac{\beta_0 S r_0^{new}}{2kT}} - e^{-\frac{\beta_0 S r_0^{min}}{2kT}} \right] \tag{67}$$

**Assumption 6**: The exponential arguments in equation are small enough so that they can be expanded. According to [3] this approximation is in some question, since the model parameters are not fixed nor well known. The true distribution is unknown. According to this assumption equation 67 becomes:

$$(N_{actual} - N_{safe}) \approx \frac{N_0 \beta_0 S r_0^{min}}{2kT} \left( 1 - \frac{r_0^{new}}{r_0^{min}} \right) \tag{68}$$

Substituting 68 in 66 and rewriting this a bit results in:

$$\frac{N_0 \beta_0 S}{2kT} r_0^{min} \left( \frac{1}{r_0^{new}} - \frac{1}{r_0^{min}} \right) \left( t_D + \frac{1}{k} \right) P_{ss}^{new} - \alpha V_{crit} \frac{1}{r_0^{new}} = 0 \tag{69}$$

The radii $r_0^{new}$ and $r_0^{min}$ can now be replaced using the VPM equations (rewriting 38-42 and 60):

$$r_0^{min} = 2 \frac{\gamma_c - \gamma}{\beta_0} \tag{70}$$

$$\frac{1}{r_0^{min}} = \frac{\gamma_c \left( P_{ss}^{min} - P_{crush} \frac{\gamma}{\gamma_c} \right)}{2\gamma (\gamma_c - \gamma)} \tag{71}$$

$$\frac{1}{r_0^{new}} = \frac{\gamma_c \left( P_{ss}^{new} - P_{crush} \frac{\gamma}{\gamma_c} \right)}{2\gamma (\gamma_c - \gamma)} \tag{72}$$

$P_{crush}$ is here $P_m - P_0$. The equations 70-72 apply to the permeable region. Applying them to the impermeable region results in an acceptable error of only 3% for values of $P_{crush}$ below 10 bar. Substituting $r_0^{min}$, $r_0^{min-1}$ and $r_0^{new-1}$ in the relation 69 by the equations given in 70-72 results in:

$$\frac{N_0 (\gamma_c - \gamma) S}{kT} \left( P_{ss}^{new} - P_{ss}^{min} \right) P_{ss}^{new} \left( t_D + \frac{1}{k} \right) - \alpha V_{crit} \left( P_{ss}^{new} - P_{crush} \frac{\gamma}{\gamma_c} \right) \approx 0 \tag{73}$$

Rewriting this leads to the quadratic equation:

$$aP_{ss}^{new\,2} - bP_{ss}^{new} + c = 0 \tag{74}$$

$$a = 1 \tag{75}$$

$$b = P_{ss}^{min} + \frac{\lambda\gamma}{\gamma_c \left(t_D + \frac{1}{k}\right)} \tag{76}$$

$$c = \left(\frac{\gamma}{\gamma_c}\right)^2 \frac{\lambda P_{crush}}{t_D + \frac{1}{k}} \tag{77}$$

Equation 56-58 is the solution of equation 74-77, where:

$$\lambda = \frac{\alpha V_{crit}\gamma_c kT}{\gamma N_0 \left(\gamma_c - \gamma\right) S} \tag{78}$$

**Parameter values**

The Yount article [3] reports the following parameter values:

| Parameter | Value |
|-----------|-------|
| $\gamma$ | 17.9 dyn/cm = 0.179 N/m |
| $\gamma_c$ | 257 dyn/cm = 2.57 N/m |
| $\tau_R$ | 20160 min |
| $r_0^{min}$ | 80 $\mu m$ |
| $\lambda$ | 7500 fsw min = 250 bar min |

**Some adaptation to VPM**

In the Yount/Maiken/Baker article [4] VPM is applied to reverse dive profiles ($P_{ss} < P_{crush}$). Some adaptations are made to the VPM.

First, the descent is not assumed to be instantaneous but takes place at a certain rate. During descent gas is loaded into the tissue compartments, leading to a smaller $P_{crush}$ value than on instantaneous descent (no gas loading). This effects the faster tissues more than the slower ones. Using the Schreiner equation one can derive a new, more general version of equation 55 for compartment j:

$$P_{ss}^{new}{}_j = \frac{2\gamma\left(\gamma_c - \gamma\right)}{\gamma_c r_0^{min}{}_j} + \frac{\gamma}{\gamma_c}\Delta_j \tag{79}$$

The effects of nuclear regeneration have not been taken into account in this equation. In this equation the set of effective crushing pressures $\Delta_j$ is given by

$$\Delta_j = P_{crush}\left(1 - Q_{N2}\right) + \frac{Q_{N2}R_c}{k_j}\left(1 - e^{-k_j t_c}\right) \tag{80}$$

In this equation is $Q_{N2}$ the Nitrogen fraction in the breathing gas mixture and $R_c$ is the crushing change rate of the partial Nitrogen pressure. In the case of rapid descent, where in the limit $t_c$ approaches 0, $R_c t_c$ approaches $P_{crush}$ and $\Delta_j$ goes to $Pcrush$. This results in the original equation 25.

In the Yount/Maiken/Baker article [4] equations 56-58 has been replaced by:

$$P_{ss}^{new} = \frac{1}{2}\left[b + \left(b^2 - 4c\right)^{1/2}\right] \tag{81}$$

where

$$b = P_{ss}^{min}{}_j + \frac{\lambda\gamma}{\gamma_c\left(t_D + \frac{1}{k_j}\right)} - \frac{\left(P_{tj}^{dive} - P_m\right)t_D}{2\left(t_D + \frac{1}{k_j}\right)} \tag{82}$$

$$c = \left(\frac{\gamma}{\gamma_c}\right)^2 \frac{\lambda P_{crush}}{t_D + \frac{1}{k_j}} - \frac{P_{ss}^{min}{}_j\left(P_{tj}^{dive} - P_m\right)t_D}{2\left(t_D + \frac{1}{k_j}\right)} \tag{83}$$

In this equation $P_{tj}^{dive}$ denotes the set of compartment tissue tensions. The last terms have been added to b and c, compared to equations 56-58. These terms become zero for saturated, not-metabolizing systems, where $P_{tj}^{dive}$ is $P_t \approx P_m$.

## The Reduced Gradient Bubble Model

Bruce Wienke extended the VPM by incorporating repetitive diving, including multi day diving [6]. This resulted in the Reduced Gradient Bubble Model (RGBM). In this section we follow the derivation given in [6]. Following equation 63, the critical volume hypothesis states for $J$ repetitive dives becomes:

$$\sum_{j=1}^{J} (N_{actual} - N_{safe})\left[\int_0^{t_D} P_{ss}^{new}dt + \int_0^{t_s j} P_{ss}^{new}e^{-kt}dt\right] < \alpha V_{crit} \tag{84}$$

$$\sum_{j=1}^{J} (N_{actual} - N_{safe})\left[P_{ss}^{new}t_D + \int_0^{t_s j} P_{ss}^{new}e^{-kt}dt\right] < \alpha V_{crit} \tag{85}$$

In this equation $t_{sj}$ is the time of the surface interval after the $j^{th}$ dive. In [6] Wienke uses $G$ (of Gradient) instead of $P_{ss}^{new}$. Hence, we followed this 'Yount notation' so far, we keep up doing so. After the last dive no more dives are made. Hence $t_{sJ}$ goes to infinity. Rewriting equation 84 (introducing $\Delta N = N_{actual} - N_{safe}$) results in:

$$\sum_{j=1}^{J} \left[\Delta N P_{ss}^{new}\left(t_D + \frac{1}{k} - \frac{1}{k}e^{-kt_{sj}}\right)\right] + \Delta N P_{ss}^{new}\left(t_{DJ} + \frac{1}{k}\right) < \alpha V_{crit} \tag{86}$$

We now define $G_j$:

For $j = 1$:

$$\Delta N G_1 = \Delta N P_{ss}^{new} \tag{87}$$

For $j = 2..J$:

$$\Delta N G_j\left(t_{Dj} + \frac{1}{k}\right) = \Delta N P_{ss}^{new}\left(t_{Dj} + \frac{1}{k}\right) - \Delta N P_{ss}^{new}\frac{1}{k}e^{-kt_{D(j-1)}} \tag{88}$$

Using equations 87 and 88, equation 86 now reads:

$$\sum_{j=1}^{J} \left[\Delta G_j\left(t_{Dj} + \frac{1}{k}\right)\right] < \alpha V_{crit} \tag{89}$$

One important property of $G$ is:

$$G_j \leq P_{ss}^{new} \tag{90}$$

Comparing this result to equation 63, equation 90 treats the dives as if they were independent dives. However, to account for the influence of the previous dives, reduced gradients are used for subsequent dives. The reduced gradients can be written as:

$$G_j = \xi_j P_{ss}^{new} \tag{91}$$

with

$$0 \leq \xi_j \leq 1 \tag{92}$$

We will look at the factors that influence $\xi$. However, [6] only shows the resulting equations, not the derivation.

**Regeneration**

During the surface intervals bubble sizes regenerate.

$$\eta_j^{reg} = \frac{\Delta N\left(t_{j-1}^{cum}\right)}{\Delta N} = e^{-\tau_R t_{j-1}^{cum}} \tag{93}$$

**Reverse diving profiles**

$$\eta_j^{exc} = \tag{94}$$

**Repetitive dives**

$$\eta_j^{rep} = \tag{95}$$

$$\xi_j = \eta_j^{reg} \eta_j^{exc} \eta_j^{rep} = \frac{(\Delta N)_{max}}{(\Delta N)_j} \left[ 1 - \left( 1 - \frac{G^{min}}{P_{ss}^{new}} \right) e^{-\tau_M t_{j-1}} \right] e^{-\tau_R t_{j-1}^{cum}} \tag{96}$$

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